Modelling induced polarization effects in frequency-domain data

Evgeny Karshakov
Institute of Control Sciences
65, Profsoyuznaya Street,
Moscow, 117997, Russia
karshakov@ipu.ru

Dmitry Khliustov*
Institute of Control Sciences
65, Profsoyuznaya Street,
Moscow, 117997, Russia
hlustov.d@gmail.com

SUMMARY

Induced polarization (IP) effects may have significant impact on airborne electromagnetic (AEM) data. They lead to dependence of apparent resistivity on the frequency of the signal. The classic approach to modelling IP consists in deriving analytical models of frequency dependent resistivity of each layer of the model. However, the amount of parameters for such model grows fast with the number of layers. Hence the problem of numerical inversion becomes intractable due to high dimensionality and ill conditioning.

This work suggests an approach to overcoming this problem. We show that the effects of IP are concentrated in relatively small number of layers and propose a simple algorithm for finding them. The results of inverting real data showing strong IP are presented.

Key words: inversion, airborne electromagnetics, frequency domain, Cole-Cole model.

INTRODUCTION

Over the past few years the induced polarisation (IP) effects have become a subject of intensive research in the field of airborne electromagnetics (AEM) data processing (Kaminski and Viezzoli, 2016). The reason for it is that in many practical applications not taking IP into account leads to problems in fitting parameters of the model.

There exist several explanations for this phenomenon. One point of view is that IP should be attributed to properties of minerals under study. Several works are devoted to construction of materials which possess the property of charge conservation (Gurin *et al.*, 2019). The experiments conducted in laboratory suggest that in the presence of inhomogeneity (such as granular structure) the environment may show frequency-dependent conductivity. Another point of view is that IP effects are caused by purely geometric properties of the surface.

One of the main practical indicators of IP is the presence of negative response in time domain or, equivalently, negative in-phase response in frequency domain (Karshakov and Moilanen, 2019). In the first part of our work we show that IP model based on Cole-Cole approach can produce results having this property. After this, we derive an inversion methodology which provides a compromise between number of parameters and explanatory power. Finally, we show results of fitting this model to real data and compare them to non-chargeable model.

All computations are done in the frequency domain. Parameters of the system (frequencies, typical altitudes, relative position of emitter and receiver) correspond to those of a real AEM system EQUATOR (Moilanen *et al.*, 2013).

METHOD AND RESULTS

Model formulation

In problems which require computing equivalent resistivity it is customary to use horizontally layered model (Zhdanov, 2009). This simplification leads to reduced amount of computations while providing good explanatory power. The model enables one to derive the response to the field of vertical magnetic dipole explicitly. Namely, for a given frequency ω the vertical component of the response is

$$H_z(\omega) = \int_0^\infty u(n_0, z, h_T, \omega) J_0(n_0 r)\, n_0^2 dn_0,$$

where J_0 is the zero-order Bessel function of the first kind, r is the horizontal shift of the receiver with respect to the dipole axis, h_T is the altitude of the dipole above the ground, z is the altitude of the receiver. Here u is the two-dimensional spectrum of the potential of the secondary field:

$$u(n_0, z, h_T, \omega) = \frac{M \exp\left(-n_0 (z+h)\right)}{2} \frac{n_1 - n_0 R^*}{n_1 + n_0 R^*},$$

where M is the amplitude of the dipole moment and R^* is the reduced spectral impedance of the medium. For K layers it is given by the formula

$$R^* = \tanh\left(n_1 h_1 + \tanh^{-1}\left(\frac{n_1}{n_1} \tanh\left(n_2 h_2 + \cdots \left(n_{K-1} h_{K-1} + \tanh^{-1} \frac{n_{K-1}}{n_K}\right) \ldots\right)\right)\right),$$

for $n_k = \sqrt{n_0^2 - \frac{i\omega\mu_0}{\rho_k}}, Re\ n_k > 0.$

In the above formula $\mu_0 = 4\pi \times 10^{-7}$ H/m the magnetic permittivity, ρ_k is the resistivity of k'th layer.

The possible approach to solving the inverse problem (that is, estimating resistivities and thicknesses of the layers) is the Kalman filter. The presence of non-linearities advocates the use of variants of Kalman approach, such as Extended and Iterated filters (Karshakov, 2020). Since these methods rely on gradient approximation, there are no easy-to-check theoretical guarantees of convergence. Hence in order to analyse real data, one often has to run the algorithm several times with different initial conditions and hyperparameters.

The formulas above are derived through explicit analysis of Maxwell equations. In the presence of environment inhomogeneous with respect to vertical coordinate only, equations can be uncoupled and reduced to second-order linear ones. Therefore, from mathematical point of view, ρ_k can be complex or even frequency dependent (as soon as we produce computations in spectral domain).

The effect of IP is usually modelled by introducing resistivity of a special form, governed by Cole-Cole equations (Cole and Cole, 1941, Pelton *et al*., 1978):

$$\rho = \rho_0\left(1 - m\left(1 - \frac{1}{1+(i\omega\tau)^c}\right)\right),$$

where ρ_0 is DC current resistivity, m is chargeability constant, τ is a relaxation time, c is a phase constant. Equations of this form first occurred in papers devoted to slow electromagnetic processes (such as electrochemistry), but later were applied for modelling other phenomena, including AEM data processing. It should be noted that the Cole-Cole equation is not the only one used in literature (see Dias (2000) for excellent review), but other models include more parameters, which makes computation problem intractable in practice.

Another thing to notice is that the function $(i\omega\tau)^c$ has several branches, and hence we must choose among a set of possible values. Indeed, $i = \exp\left(i * \frac{\pi}{2}\right) = \exp\left(i * \left(\frac{\pi}{2} + 2\pi k\right)\right)$ for integer k. Hence after raising it to the power of c we obtain $\exp\left(i\frac{c\pi}{2}\right)$, $\exp\left(i\frac{5c\pi}{2}\right)$ ... as possible values, all distinct as soon as c and π are incommensurable. We further remark on this issue in the next section.

Theoretical considerations

As it was already mentioned, one of the main practical indicators of IP in frequency domain is the presence of negative in-phase response. We tried to obtain it in simulation by posing optimisation problem $Re\ H_z(\omega) \to min$ and solving it over ω and resistivity parameters. It turned out that in order for the problem to have negative solution one must have $Re\ \rho_k > 0, Im\ \rho_k > 0$ for resistivity of at least one of the layers. However, the vanilla Cole-Cole model with $i^c = \exp\left(i\frac{c\pi}{2}\right)$ is uncapable of producing such resistivity. Hence we have chosen another branch of the function, taking $i^c = \exp\left(i\frac{5c\pi}{2}\right)$. The results of simulating half-space model with parameters $\rho = 1000\ \Omega m, m = 0.5, \tau = 0.001\ s, c = 0.5$ are presented in figure 1. There we present three curves for each graph: quadrature (Im) and inphase (Re) components in frequency domain and off-time signal in time-domain. Index "IP0" is related to the branch number 0 with $i^c = \exp\left(i\frac{c\pi}{2}\right)$, "IP1" is related to the branch number 1 with $i^c = \exp\left(i\frac{5c\pi}{2}\right)$, "noIP" is related to $\rho = \rho_0$.

However, it is not clear why we should choose this branch of power function instead of any other. One of the possible approaches would be to introduce additional phase parameter in Cole-Cole formula and search for branch of the form $\exp(i\phi)$. But this leads to functions with singularities in the domain and is a subject of further investigation.

If the model has all resistivities determined by Cole-Cole formula, the number of parameters is high. Hence one needs an additional regularization in order for the inverse problem to be tractable. We adopted a hypothesis that chargeability is determined by relatively small number of layers (1-3). To find these layers, we fitted a model with one chargeable layer and non-chargeable others. After varying the depth in which chargeability was located, we could obtain residuals between model prediction and measured response. For some depths, the residual turned out to be several times lower (30-40 as compared to 150-200) than for others. Although it was still too high to consider any of obtained result a good fit, this procedure enabled us to make a choice as to which layers should be chargeable.

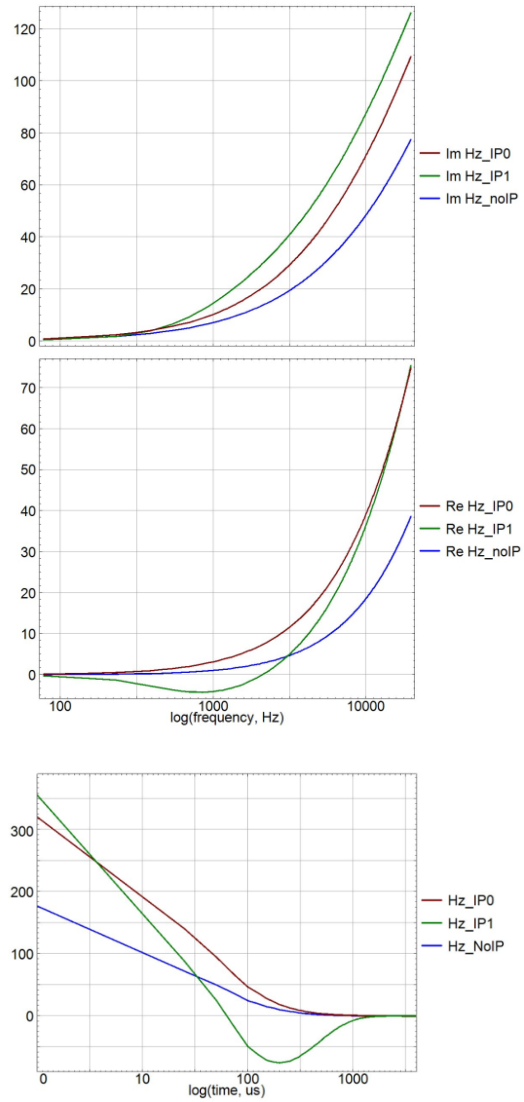

Figure 1. The response of half-space model in frequency (quadrature and inphase) and time domains

This simple algorithm is valid only for small sections of data, since the actual profile can change significantly on large scales. When the residual of final model becomes too high, one needs to repeat the procedure and determine new depths of chargeable layers.

Main results

Here we provide results of real data processing based on our approach. The data consisted of responses for 15 frequencies ranging from 77 to 15000 Hz, with in-phase and quadrature components measured for each. We used model with 25 layers with thickness of i'th layer equal to $4 * 1.1085^i$ meters. The first step of the algorithm consisted in fitting the model which had frequency dependent resistivity in one of 25 layers. An example of relative discrepancy (determined by formula

$\left(\sum_{freqs} \frac{(response_{estimated}(freq) - response_{true}(freq))^2}{\sigma^2_{noise}(freq)}\right)^{\frac{1}{2}}$ is presented in figure 2.

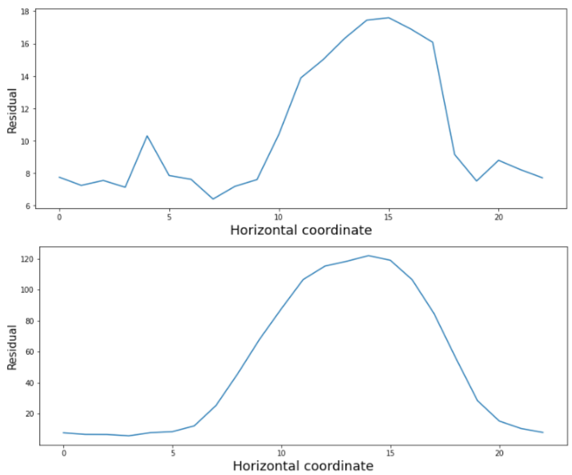

Figure 2. The residual of model with one chargeable layer (a) Chargeability in layer # 2(b) Chargeability in layer # 5 The second graph clearly exceeds the first

It is easy to see that by locating chargeability in layer #2 we decrease residual significantly compared to locating it in layer #5.

It turned out that in order to decrease the residual one must locate chargeability in layers 2, 4, 6 and 7. In order to further decrease the number of parameters we changed consecutive layers 6 and 7 to one layer with thickness equal to the sum of respective thicknesses. Hence we had to fit a model with 33 parameters. The resulting relative discrepancy is given in figure 3. It is easy to see that residual does not exceed 10, which may be considered a good fit.

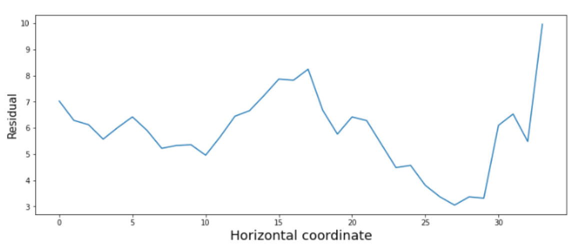

Figure 3. The residual of the final model

Figure 4 gives values of Cole-Cole parameters for each of the layers obtained by model.

Figure 5 shows the results of data inversion. The survey was carried out in Siberia, in the permafrost region. The melting zone is considered to be the main source of chargeability. We would like to point out the following features. First, after applying the chargeable model for the inversion, we see horizontally continuous layers. Second, even in the case of positive inphase responses (left part), the chargeability model provides more adequate solution according to known local geological properties: the lower conductive layer is presented along the whole flight trajectory.

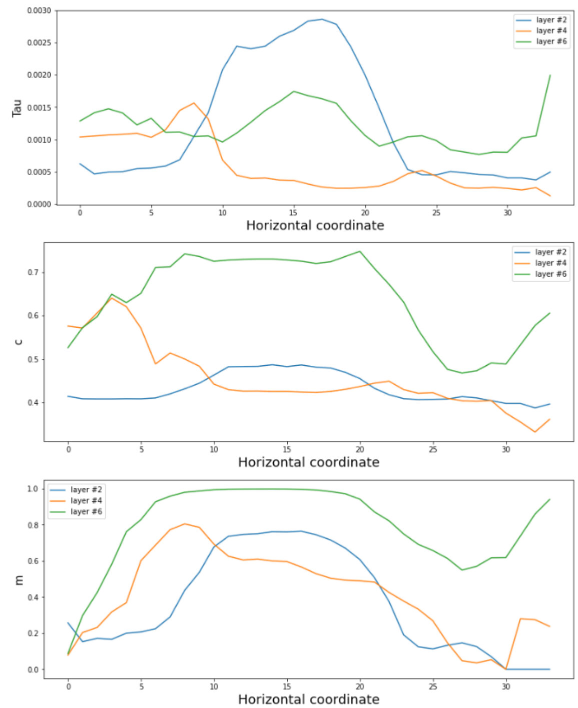

Figure 4. The Cole-Cole parameters: time constant, exponent and chargeability

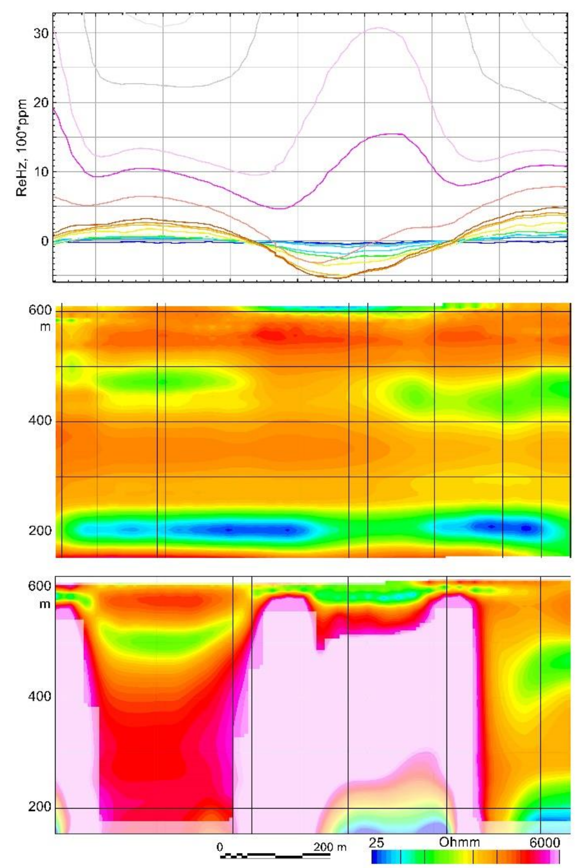

Figure 5. The inphase response curves and inversion results for the models with (top) and without (bottom) chargeability.

CONCLUSIONS

In this work we studied the problem of inverting AEM data in the presence of IP effects. We considered horizontally layered one dimensional model with frequency dependent resistivity, given by Cole-Cole formula. It has been shown that IP effects are mostly determined by local properties of environment and are concentrated in a small number of layers. Based on this, we suggested an approach to choosing the depth of chargeable layers by fitting several simpler models and analysing residuals.

For a demonstration of our approach, we analysed real data by estimating parameters of our model. We used dataset obtained by the EQUATOR AEM system. The environment demonstrates signs of IP presence, most likely connected to ice melting.

Further research directions include formalizing the procedure of chargeable layer identification. Some additional investigation should be conducted on choosing the appropriate branch of complex-valued resistivity function, discussed above.

ACKNOWLEDGMENTS

We would like to thank Dr. A. Volkovitsky for a fruitful discussion of the chargeability model, and Dr. J. Moilanen for help with result visualization.

REFERENCES

Cole, K.S., and R.H. Cole, 1941. Dispersion and absorption in dielectrics I. Alternating current characteristics. *The Journal of chemical physics* 9(4): 341-351.

Dias, C., 2000. Developments in a model to describe low-frequency electrical polarization of rocks. *Geophysics* 65(2): 437-451.

Gurin, G., Titov, K., and Y. Ilyin, 2019. Induced polarization of rocks containing metallic particles: evidence of passivation effect. *Geophysical Research Letters* 46(2): 670-677.

Kaminski, V., and A. Viezzoli, 2017. Modeling induced polarization effects in helicopter time-domain electromagnetic data: Field case studies. *Geophysics* 82(2): B49-B61.

Karshakov, E., and J. Moilanen, 2019. Overcoming Airborne IP in Frequency Domain: Hopes and Disappointments. *SAGA Biennial Conference & Exhibition*: 1-4.

Karshakov, E., 2020. Iterated extended Kalman filter for airborne electromagnetic data inversion. *Exploration Geophysics* 51(1): 66-73.

Moilanen, J., Karshakov, E., and A. Volkovitsky, 2013. Time-domain helicopter EM System "Equator": resolution, sensitivity, universality. *13th SAGA biennial and 6thInternational AEM conference AEM-2013*, Expanded Abstracts: 1-4.

Pelton, W.H., Ward, S.H., Hallof, G., Sill, W.R., and P.H. Nelson, 1978. Mineral discrimination and removal of inductive coupling with multifrequency IP. *Geophysics* 43(3): 588-609.

Zhdanov, M. S., 2009. Geophysical electromagnetic theory and methods. Elsevier.